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A Model of the Fatigue Life Distribution of Composite Laminates Based on Their Static Strength Distribution

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Abstract

The reasons of the static strength dispersion and the fatigue life dispersion of composite laminates are analyzed in this article. It is concluded that the inner original defects, which derived from the manufacturing process of composite laminates, are the common and major reason of causing the random distributions of the static strength and the fatigue life. And there is a correlative relation between the two distributions. With the study of statistical relationship between the fatigue loading and the fatigue life in the uniform confidence level and the same survival rate $S-N$ curves of material, the relationship between the static strength distribution and the fatigue life distribution through a material $S-N$ curve model has been obtained. And then the model which is used to describe the distributions of fatigue life of composites, based on their distributions of static strength, is set up. This model reasonably reflects the effects of the inner original defects on the static strength dispersion and on the fatigue life dispersion of composite laminates. The experimental data of three kinds of composite laminates are employed to verify this model, and the results show that this model can predict the random distributions of fatigue life for composites under any fatigue loads fairly well.

Keywords: composite; static strength; fatigue life; $S-N$ curve; random distribution

1 Introduction

Composite laminates are widely used in the areas of aerospace, transportation and construction engineering because of their high specific stiffness and strength. A great deal of research has been conducted to improve the properties of composite materials. In engineering applications, the static strength and fatigue life of composite laminates are important property parameters, which are usually statistical values and obtained from a mass of experiments. To reduce experiment time and lessen experiment costs, it is helpful to study the relationship between static strength distribution and fatigue life distribution of composite materials.

There are many types of inner original defects in composite laminates^[1], for example air bubble, void, inclusion, resin-starved area, resin-rich area and fiber bend. When fiber content and ply stacking sequence of composite laminate are prearranged, the random distribution of defects will affect the crack forming, propagation and growth rate, which are the major factors affecting the static strength dispersion. Under cycling load, four basic fatigue failure modes will occur in composite laminate^[2], they are matrix cracking, interfacial debonding, delamination and fiber breakage. The location, type and size of the original defects and the component content of composites will deeply affect the fatigue failure mode and the fatigue life value for the given fatigue load. Therefore, the random distribution of original defects also affects the fatigue life dispersion of com-

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posite laminates.

The random distribution of original defects is the major factor that affecting the dispersion of static strength and fatigue life of composites. In other words, the dispersion of static strength and fatigue life come from the original defect dispersion. Therefore, there is certainly a relationship between static strength statistical distribution and fatigue life statistical distribution in composite laminates. This relationship is investigated by analyzing the statistic characteristics of static strength and fatigue life of laminates. Yang^[3-4] presented a model to describe the relationship by fitting experimental data. However, the fatigue life curve ($S-N$ curve) model cited in his model is only suitable to describe one region of $S-N$ curve of material. Caprino^[5-9] used a $S-N$ curve model to describe this relationship. However, the model is also not able to describe the $S-N$ curve of materials in the whole region. Ref.[10] provided a $S-N$ curve model, which can describe all regions of material $S-N$ curve. Based on the $S-N$ curve model, a model is presented to describe the relationship between static strength distribution and fatigue life distribution in this article. Three sets of experimental data are employed to verify this model, and the results show that the model describes the relationship fairly well.

2 Statistical Distribution Model

Under cyclic load, the non-inverse structural changes will occur in the micro local field or macro structure of materials. These changes produce the generalized monotonic decreasing characteristics of material $S-N$ curve, as shown in Fig 1. The failure probability of the specimen being subjected to N fatigue cycles for given fatigue strength σ_{\max}^* and the failure probability of the specimen being subjected to fatigue strength σ_{\max} for given fatigue life N^* are same with the same confidence level. The failure probabilities for different assigned fatigue lives are also the same, because the $S-N$ curves of materials are derived from the test results with the same survival rate. Therefore, the failure probabilities of the static strength and the fatigue life for dif-

ferent fatigue load are the same^[11-12].

The failure probabilities of the static strength σ^* and the fatigue life N^* for given fatigue load σ_{\max}^* are the same for the uniform confidence level and the same survival rate $S-N$ curve of composite laminates, which can be expressed as

$$\int_0^{\sigma^*} f(\sigma) d\sigma = \int_0^{\sigma_{\max}^*} h(\sigma_{\max} | N) d\sigma = \int_0^{N^*} g(N | \sigma_{\max}) dN \quad (1)$$

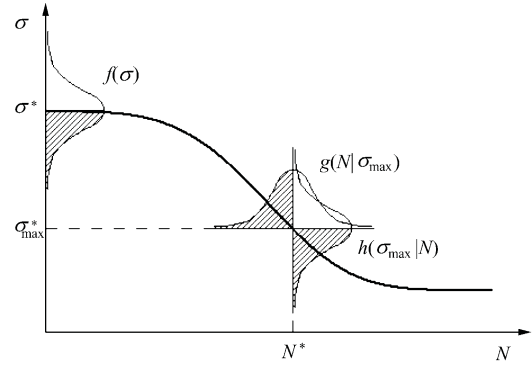


Fig.1 Fatigue life curve.

It is reasonable^[13-16] to assume that the statistical distribution of the static strength σ_{ult} follows a two-parameter Weibull distribution

$$F_{\sigma_{\text{ult}}}(X) = P\{\sigma_{\text{ult}} \leq X\} = 1 - \exp\left(-\left(\frac{X}{\beta}\right)^{\alpha}\right) \quad (2)$$

where $F_{\sigma_{\text{ult}}}(X)$ denotes the failure probability when the specimen is subjected to a static load X , σ_{ult} the static strength, α the shape parameter, and β the scale parameter. The parameters α and β can be calculated by statistical method

$$\bar{X} = E(X) = \beta \Gamma\left(1 + \frac{1}{\alpha}\right) \quad (3a)$$

$$S^2 = D(X) = \beta^2 \left\{ \Gamma\left(1 + \frac{2}{\alpha}\right) - \left[\Gamma\left(1 + \frac{1}{\alpha}\right) \right]^2 \right\} \quad (3b)$$

where \bar{X} and S^2 are the mean and variance of the experimental sample, respectively, and $E(X)$ and $D(X)$ the mean and variance of random variable, respectively. The shape parameter α describes the failure mechanism of laminates. The larger the parameter α is, the less the variation coefficient or dispersion of random variable X is. The scale parameter β is also the characteristic strength of random variable X .

The author had presented a S - N curve model of composite laminates^[10] as

$$\frac{\sigma_{\max}}{\sigma_{\text{ult}}} = 1 + m \left(\exp \left(- \left(\frac{\lg N}{b} \right)^a \right) - 1 \right) \quad (4)$$

where σ_{\max} is the maximum stress and parameters a , b and m the experimental constants. Based on Eq.(1), the failure probability of fatigue life of composite laminate can be expressed as

$$F_N(n) = P\{N \leq n\} = P\{\sigma_{\text{ult}} \leq X\} \quad (5)$$

where $F_N(n)$ denotes the failure probability of the specimen being subjected to n fatigue cycles. Substituting Eq.(4) and Eq.(5) into Eq.(2) yields

$$F_N(n) = 1 - \exp \left[- \left(\frac{\sigma_{\max}}{\beta \left(1 + m \left(\exp \left(- \left(\frac{\lg N}{b} \right)^a \right) - 1 \right) \right)} \right)^\alpha \right] \quad (6)$$

Eq.(6) describes the relationship between static strength distribution and fatigue life distribution. When the parameters α , β , a , b and m are obtained from experiments, the failure probability of fatigue life of composite laminate for any given fatigue load can be calculated by Eq.(6).

Eq.(4) can be rewritten as

$$\sigma_* = \sigma_{\text{ult}} = \frac{\sigma_{\max}}{1 + m \left(\exp \left(- \left(\frac{\lg N}{b} \right)^a \right) - 1 \right)} \quad (7)$$

When the experimental data of fatigue life are obtained, the estimated static strength σ_* can be calculated by Eq.(7). A comparison between the estimated static strength statistical distribution obtained by Eq.(7) and the test results can be made. Then, the rationality of Eq.(6) that describes the relationship between the static strength distribution and the fatigue life distribution of composite laminates can be verified by the comparison.

3 Verification

The data cited from Refs.[13-15] are used to validate the presented model. The empirical fre-

quencies of static strength or fatigue life of laminates are calculated by

$$p_i = \frac{i}{k+1} \quad (8)$$

where k is the sample size of experimental data and the arrangement sequence of experimental data is monotone increasing, and i the i th number of the sample. Refs.[13-15] provide 25 items of static strength experimental data of T300/934 [0/45/90/-45₂/90/45/0]₂, 15 items of static strength experimental data of T300/5208 [± 45]_{2S} and 15 items of static strength experimental data of T300/5208 [90/ ± 45 /0]_S, respectively. Based on these experimental data, the parameter α and β of the static strength distribution function can be calculated by Eq.(3) and the empirical frequencies of the static strength can be calculated by Eq.(8). Then, the correlation coefficient R^2 between the empirical frequencies and the functional values of Eq.(2) can be calculated. The calculated results are presented in Table 1. According to these results, the two-parameter Weibull function describes the statistical distribution of static strength fairly well, as shown in Fig 2.

Table 1 Parameter values of the statistical distribution of static strength

Laminate	α	β	R^2
[0/45/90/-45 ₂ /90/45/0] ₂ ^[13]	25.099	70.641	0.972 3
[± 45] _{2S} ^[14]	43.357	14.441	0.931 1
[90/ ± 45 /0] _S ^[15]	25.156	88.787	0.903 7

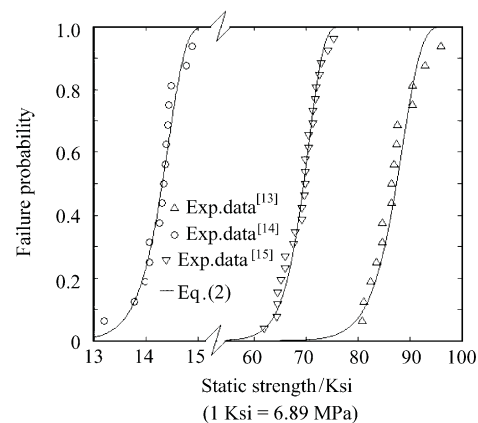


Fig.2 Distribution of static strength.

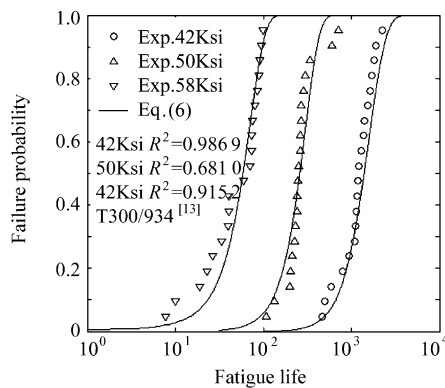
Refs.[13-15] also provide experimental data of fatigue life with stress ratio($R=\sigma_{\min}/\sigma_{\max}$) of 0, 0.1, and 1/36 for three kinds of laminates, respectively. Then, Eq.(4) that is the S - N curve of laminate can be obtained from the static strength data and the fatigue life data. And the correlation coefficient R^2 between the empirical frequencies of experimental data and the functional values of Eq.(4) can be gotten. The calculated results are presented in Table 2.

Table 2 Parameter values of the S - N curve of laminates

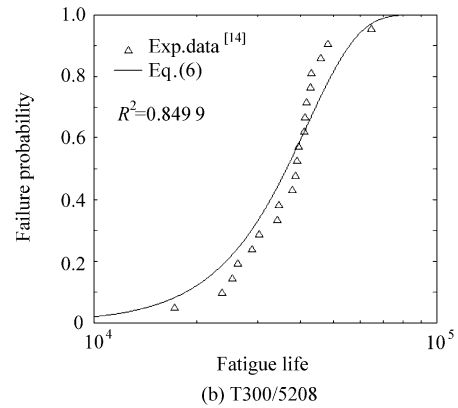
Laminate	a	b	m	R^2
[0/45/90/-45/90/45/0] ₂ ^[13]	2.216	5.539	0.54	0.991 7
[±45] _{2S} ^[14]	1.212	7.314	1.0	0.971 3
[90/±45/0] _S ^[15]	1.816	8.097	1.0	0.975 2

Ref.[13] provides 20 items of fatigue life experimental data under the maximum stresses 42, 50, and 58 Ksi, respectively. Refs.[14-15] provide 20 items of fatigue life experimental data under the maximum stresses of 8.134 Ksi and 60.57 Ksi, respectively. Based on these experimental data, the empirical frequencies and the failure probabilities of fatigue life of laminates are calculated by Eq.(8) and Eq.(6), respectively. Then, the correlation coefficient R^2 between the empirical frequencies and failure probabilities can be gotten, as shown in Fig 3.

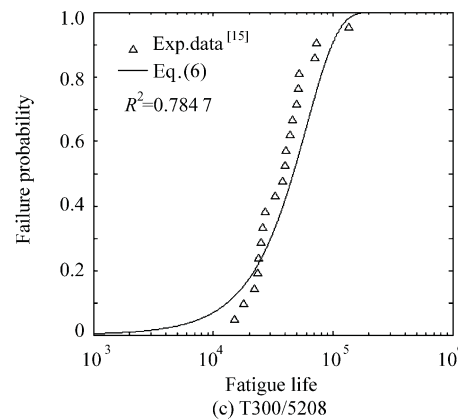
The statistical distribution of the estimated static strength can be calculated by Eqs.(7)-(8) utilizing 69 items of fatigue life experimental data of T300/934 [0/45/90/-45/90/45/0]₂^[13], 47 items of fatigue life experimental data of T300/5208[±45]_{2S}^[14],



(a) T300/934



(b) T300/5208



(c) T300/5208

Fig.3 Distribution of fatigue life.

and 45 items of fatigue life experimental data of T300/5208 [90/±45/0]_S^[15]. The comparison between the statistical distribution of estimated static strength and the statistical distribution of static strength is presented in Fig.4. The results show that Eq.(6) can properly describe the relationship between static strength distribution and fatigue life distribution of composite laminates.

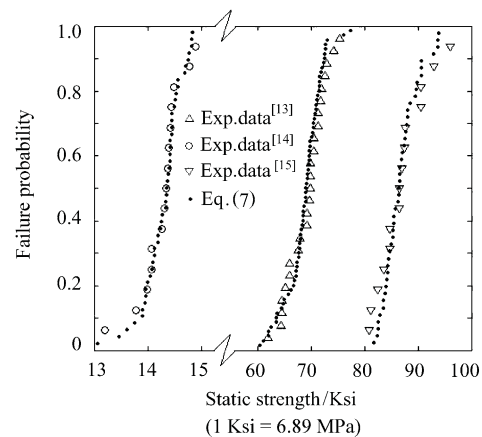


Fig.4 Comparison of the distributions of estimated static strength and test static strength.

From the calculated results of three kinds of laminates experimental data, it is concluded that the two-parameter Weibull function can be used to describe the statistical distribution of static strength and when the static strength distribution and the $S-N$ curve of laminate are obtained, Eq.(6) can be used to predict the distribution of fatigue life of composite laminates for different fatigue loading.

4 Discussion and Conclusions

The location, size and type of the inner original defects are derived from the manufacturing process of composite laminates. The distribution of defects is a random original distribution and it is the major factor to form the static strength dispersion and fatigue life dispersion of laminates. Therefore, the distribution form, dispersion degree and change trend of the static strength dispersion and the fatigue life dispersion are consistent, and their distribution functions are similar. The way of loading of the static strength experiment of composite laminate is monotonously increasing the load until the specimen being failed. The way of loading of fatigue experiment of composite laminate is the specimen being subjected to cyclic load until it being failed. The cyclic load is less than the static strength of laminate. The differences of loading ways determine the differences of crack propagation rates and ways, and lead to the differences of the static strength dispersion and the fatigue life dispersion of composite laminates. Therefore, their distribution functions can not uniquely determine each other. For the $S-N$ curve of composite laminate with uniform confidence level and same survival rate, the failure probability of the specimen being subjected to static load is equal to the failure probability of the specimen being subjected to fatigue cycles. Then, the static strength distribution function and the fatigue life distribution function can be integrated with the $S-N$ curve model of composite laminate.

The statistical distributions of the static strength and the fatigue life obey the statistical "weak link" theory of materials. Theoretically, they follow the Weibull distribution. The static strength

distribution of laminate is described by a two-parameter Weibull function in this article. When a $S-N$ curve model that can predict the fatigue life of material in whole region is employed to integrate the distributions, a model used to predict the fatigue life distribution of laminate is then set up. The model is Eq.(6). Eq.(6) approximately follows a two-parameter Weibull distribution and its shape parameter approximates the parameter α . The parameter α is obtained from static strength experiment and it involves the effects of original defects on fatigue life dispersion. However, the effect of fatigue loading way on fatigue life dispersion is not involved in parameter α . It is the major reason that Eq.(6) is not able to accurately predict the fatigue life dispersion of laminates.

To analyze the factors influencing the dispersion of static strength and fatigue life, it is necessary to do more experiments. However, this approach is infeasible in engineering application. The relationship between the static strength dispersion and the fatigue life dispersion is analyzed in this article. Then, an effective model is presented to describe the fatigue life distribution. Several sets of validated results show that the present model can be used to predict the laminate fatigue life distribution for any given fatigue load.

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